

SYLLABUS FOR MATHEMATICAL SCIENCES (FGT)

Matrices & Determinant:

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists.

Determinant of a square matrix (up to 3×3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle.

Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equation by examples, solving system of linear equations in two or three variables using inverse of a matrix. Rank and determinant of matrices, linear equations. Eigen values and eigen vectors, Cayley-Hamilton theorem

Elementary set theory, Sets:

Sets

Sets and their representations. Empty set, Finite & Infinite sets, Equal sets, Subsets, Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set, Properties of complement sets. Some basic notation of set theory, countable and uncountable sets. Supremum and infimum of a set. Elements of point set theory including properties of open, closed and compact sets in

Sequence and series:

Sequence and Series, Arithmetic Progression (A.P), Arithmetic Mean (A.M), Geometric Progression (G.P), general term of a G.P, sum of n terms of a G.P. Arithmetic and Geometric series, infinite G.P. and its sum. Geometric mean (G.M), relation between A.M and G.M, Sum to n term of the special series $\sum n$, $\sum n^2$ and $\sum n^3$. Convergence, Sequences and series of functions, uniform convergence.

Binomial Theorem:

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications. Bolzano Weierstrass theorem, Heine Borel theorem.

Continuity and Differentiability:

Continuity, uniform continuity. Continuity and Differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit function.

Concepts of exponential and logarithmic functions. Derivatives of $\log e^x$ and e^x . Logarithmic differentiation. Derivative of functions expressed in parametric forms.

Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents and normal, approximation, maxima and minima. Simple problems (that illustrate basic principles and understanding of the subject as well as real life situations).

Limits and Derivatives:

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive idea of limit, \limsup , \liminf . Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Mean value theorem.

Integrals: Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Definite integrals as a limit of a sum. Fundamental Theorem of Calculus, Basic properties of definite integrals and evaluation of definite integrals. Double and Triple Integrals, Change of variables. Applications to evaluation of areas, Volume, Centre of Gravity and Moments of Inertia etc. Change of order of integration in double integrals

Differential Equations

Exact differential equations. First order and higher degree equations solvable for x, y, p . Clairaut's form and singular solutions. Geometrical meaning of a differential equation. Orthogonal trajectories. Linear differential equations with constant and variable coefficients. Variation of Parameters method, reduction method, series solutions of differential equations. Power series method, Bessel and Legendre equations. (only series solution).

Vector spaces: Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors, scalar triple product.

Linear Inequalities:

Linear inequalities, Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables - graphically.

Subspaces, linear dependence, basis, dimension, algebra of linear transformations.

Straight Lines :

Recall of 2-D from earlier classes, Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two-point form, intercept form and normal form, General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

Conic Sections:

Sections of a cone; circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equations of a circle.

Introduction to Three-dimensional Geometry:

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Three-dimensional Geometry:

Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

Complex Numbers and Quadratic Equations:

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square-root of a Complex number.

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.

Trigonometric Functions:

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x . Signs of trigonometric functions and sketch of their graphs. Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of $\sin x$, $\sin y$, $\cos x$ & $\cos y$. Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$. General solution of trigonometric equations of the type $\sin \theta = \sin \alpha$, $\cos \theta = \cos \alpha$ and $\tan \theta = \tan \alpha$.

Limit and Continuity

Limit and Continuity of functions of two variables. Partial differentiation. Change of variables. Partial derivation and differentiability of real-valued functions of two variables. Schwartz's and Young's Theorem. Statements of Inverse and implicit function theorems and applications. Euler's theorem on homogeneous functions. Taylor's theorem for functions of two variables. Jacobians.

Permutations & Combinations:

Fundamental principle of counting, Factorial $n(n!)$ Permutations and combinations, derivation of formulae and their connections, simple applications. Pigeon-hole principle, inclusion-exclusion principle.

Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , congruences, Chinese Remainder Theorem, Euler's ϕ -function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

Numerical Analysis:

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Number Theory

The division algorithm, The greatest common divisor, least common multiple, The Euclidean algorithm, The Diophantine equation $ax + by = c$ Prime numbers and their distribution, The fundamental theorem of arithmetic, Basic properties of congruences, Linear congruences, Special divisibility tests.

Mathematical Reasoning:

Mathematically acceptable statements. Connecting words/phrases—consolidating the understanding of “if and only if (necessary and sufficient) condition”, “implies”, “and/or”, “implied by”, “and”, “or”, “there exists” and their use through variety of examples related to real life and Mathematics, Validating the statements involving the connecting words- difference between contradiction, converse and contrapositive.

Statistics:

Measure of dispersion: mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

Probability:

Random experiments: outcomes, sample spaces(set representation). Events: Occurrence of events, 'not', 'and' & 'or' events, exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability, connections with the theories of earlier classes.

Probability of an event, probability of 'not', 'and' & 'or' events. Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution. Sample space, discrete

probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems.

Linear Programming Problem:

Introduction, definition of related terminology such as constraints, objectives function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints)

Statics: Basic notation, Newton Laws of motion, system of two forces, parallelogram law of forces, resultant of two collinear forces, resolution of forces, moment of a force, couple, theorem on moments of a couple, coplaner forces, resultant of three coplanar concurrent forces, theorem of resolved parts, resultant of two forces acting on a rigid body, Varignon's theorem, generalized theorem of moments.

Equilibrium

Equilibrium of two concurrent forces, equilibrium condition for any number of coplanar concurrent forces, Lami's theorem. $\lambda - \mu$ theorem, theorems of moments, resultant of a force and a couple. Equilibrium conditions for coplanar non-concurrent forces.

Dynamics:

Motion of a particle with constant acceleration, acceleration of falling bodies, motion under gravity, motion of a body projected vertically upward, motion of a two particles connected by a string, motion along a smooth inclined plane, constrained motion along a smooth inclined plane. Variable acceleration: Simple harmonic motion, elastic string, curvilinear in a plane, Definition of velocity and acceleration, Projectile, motion in a circle, motion under constraints, central force motion